

Correction : premiers termes d'une suite

www.bossetesmaths.com

Exercice 1 (Suites explicites)

1) $u_n = 4n^2 - 10$

- $u_0 = 4 \times 0^2 - 10 = 4 \times 0 - 10 = 0 - 10 = \boxed{-10}$;
- $u_1 = 4 \times 1^2 - 10 = 4 \times 1 - 10 = 4 - 10 = \boxed{-6}$;
- $u_2 = 4 \times 2^2 - 10 = 4 \times 4 - 10 = 16 - 10 = \boxed{6}$;
- $u_3 = 4 \times 3^2 - 10 = 4 \times 9 - 10 = 36 - 10 = \boxed{26}$;
- $u_4 = 4 \times 4^2 - 10 = 4 \times 16 - 10 = 64 - 10 = \boxed{54}$.
- Le 58ème terme : $u_{57} = 4 \times 57^2 - 10 = 4 \times 3\,249 - 10 = 12\,996 - 10 = \boxed{12\,986}$.

Vérification à la calculatrice :

Plot1	Plot2	Plot3
Y1 = 4X ² - 10		
Y2 =		
Y3 =		
Y4 =		
Y5 =		
Y6 =		

X	Y1	
0	-10	
1	-6	
2	6	
3	26	
4	54	
5	90	
6	134	

X=0

X	Y1	
57	12986	
58	13446	
59	13914	
60	14390	
61	14874	
62	15366	
63	15866	

X=57

2) $u_n = (-1)^n$

- $u_0 = (-1)^0 = \boxed{1}$;
- $u_1 = (-1)^1 = \boxed{-1}$;
- $u_2 = (-1)^2 = \boxed{1}$;
- $u_3 = (-1)^3 = \boxed{-1}$;
- $u_4 = (-1)^4 = \boxed{1}$.
- Le 58ème terme : $u_{57} = (-1)^{57} = \boxed{-1}$.

Vérification à la calculatrice :

Plot1	Plot2	Plot3
Y1 = (-1) ^X		
Y2 =		
Y3 =		
Y4 =		
Y5 =		
Y6 =		

X	Y1	
0	1	
1	-1	
2	1	
3	-1	
4	1	
5	-1	
6	1	

X=0

X	Y1	
57	-1	
58	1	
59	-1	
60	1	
61	-1	
62	1	
63	-1	

X=57

3) $u_n = \frac{3-n}{n+2}$

- $u_0 = \frac{3-0}{0+2} = \frac{3}{2} = 1,5$;
- $u_1 = \frac{3-1}{1+2} = \frac{2}{3} \approx 0,667$;
- $u_2 = \frac{3-2}{2+2} = \frac{1}{4} = 0,25$;
- $u_3 = \frac{3-3}{3+2} = \frac{0}{5} = \boxed{0}$;
- $u_4 = \frac{3-4}{4+2} = \frac{-1}{6} = -\frac{1}{6} \approx -0,167$.
- Le 58ème terme : $u_{57} = \frac{3-57}{57+2} = \frac{-54}{59} = -\frac{54}{59} \approx -0,915$.

Vérification à la calculatrice :

Plot1	Plot2	Plot3
Y1 = (3-X)/(X+2)		
Y2 =		
Y3 =		
Y4 =		
Y5 =		
Y6 =		
Y7 =		

X	Y1	
0	1.5	
1	.66667	
2	.25	
3	0	
4	-.1667	
5	-.2857	
6	-.375	

X=0

X	Y1	
57	-.9153	
58	-.9167	
59	-.918	
60	-.9194	
61	-.9206	
62	-.9219	
63	-.9231	

X=57

4) $u_n = \cos\left(n \frac{\pi}{2}\right)$: faire un cercle trigonométrique pour trouver les valeurs du cosinus

• $u_0 = \cos\left(0 \times \frac{\pi}{2}\right) = \cos 0 = \boxed{1}$;

• $u_1 = \cos\left(1 \times \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = \boxed{0}$;

• $u_2 = \cos\left(2 \times \frac{\pi}{2}\right) = \cos \pi = \boxed{-1}$;

• $u_3 = \cos\left(3 \times \frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = \boxed{0}$;

• $u_4 = \cos\left(4 \times \frac{\pi}{2}\right) = \cos(2\pi) = \boxed{1}$.

• Le 58ème terme : $u_{57} = \cos\left(57 \times \frac{\pi}{2}\right)$; or $57 \frac{\pi}{2} = \frac{57\pi}{2} - 28\pi = \frac{57\pi}{2} - \frac{56\pi}{2} = \frac{\pi}{2} [2\pi]$ donc $u_{57} = \cos\left(\frac{\pi}{2}\right) = \boxed{0}$.

Vérification à la calculatrice : penser à mettre la calculatrice en Mode "Radians"; la valeur affichée pour u_{57} est proche de zéro, ne pas se fier au résultat -7×10^{-13} :

<table border="1"> <tr><td>NORMAL</td><td>SCI</td><td>ENG</td></tr> <tr><td>Float</td><td>0</td><td>1 2 3 4 5 6 7 8 9</td></tr> <tr><td>Radian</td><td>DEGREE</td><td></td></tr> <tr><td>Func</td><td>PAR</td><td>POL SEQ</td></tr> <tr><td>CONNECTED</td><td>DOT</td><td></td></tr> <tr><td>SEQUENTIAL</td><td>SIMUL</td><td></td></tr> <tr><td>REAL</td><td>a+bi</td><td>re^θi</td></tr> <tr><td>FULL</td><td>HORIZ</td><td>G-T</td></tr> <tr><td></td><td>↓NEXT↓</td><td></td></tr> </table>	NORMAL	SCI	ENG	Float	0	1 2 3 4 5 6 7 8 9	Radian	DEGREE		Func	PAR	POL SEQ	CONNECTED	DOT		SEQUENTIAL	SIMUL		REAL	a+bi	re^θi	FULL	HORIZ	G-T		↓NEXT↓		<table border="1"> <tr><td>Plot1</td><td>Plot2</td><td>Plot3</td></tr> <tr><td>Y1 = cos(X*π/2)</td><td></td><td></td></tr> <tr><td>Y2 =</td><td></td><td></td></tr> <tr><td>Y3 =</td><td></td><td></td></tr> <tr><td>Y4 =</td><td></td><td></td></tr> <tr><td>Y5 =</td><td></td><td></td></tr> <tr><td>Y6 =</td><td></td><td></td></tr> <tr><td>Y7 =</td><td></td><td></td></tr> </table>	Plot1	Plot2	Plot3	Y1 = cos(X*π/2)			Y2 =			Y3 =			Y4 =			Y5 =			Y6 =			Y7 =			<table border="1"> <thead> <tr><th>X</th><th>Y1</th><th></th></tr> </thead> <tbody> <tr><td>0</td><td>1</td><td></td></tr> <tr><td>1</td><td>0</td><td></td></tr> <tr><td>2</td><td>-1</td><td></td></tr> <tr><td>3</td><td>0</td><td></td></tr> <tr><td>4</td><td>1</td><td></td></tr> <tr><td>5</td><td>0</td><td></td></tr> <tr><td>6</td><td>-1</td><td></td></tr> </tbody> </table> <p>X=0</p>	X	Y1		0	1		1	0		2	-1		3	0		4	1		5	0		6	-1		<table border="1"> <thead> <tr><th>X</th><th>Y1</th><th></th></tr> </thead> <tbody> <tr><td>57</td><td>-7E-13</td><td></td></tr> <tr><td>58</td><td>-1</td><td></td></tr> <tr><td>59</td><td>9E-13</td><td></td></tr> <tr><td>60</td><td>1</td><td></td></tr> <tr><td>61</td><td>-1E-12</td><td></td></tr> <tr><td>62</td><td>-1</td><td></td></tr> <tr><td>63</td><td>1E-12</td><td></td></tr> </tbody> </table> <p>X=57</p>	X	Y1		57	-7E-13		58	-1		59	9E-13		60	1		61	-1E-12		62	-1		63	1E-12	
NORMAL	SCI	ENG																																																																																																				
Float	0	1 2 3 4 5 6 7 8 9																																																																																																				
Radian	DEGREE																																																																																																					
Func	PAR	POL SEQ																																																																																																				
CONNECTED	DOT																																																																																																					
SEQUENTIAL	SIMUL																																																																																																					
REAL	a+bi	re^θi																																																																																																				
FULL	HORIZ	G-T																																																																																																				
	↓NEXT↓																																																																																																					
Plot1	Plot2	Plot3																																																																																																				
Y1 = cos(X*π/2)																																																																																																						
Y2 =																																																																																																						
Y3 =																																																																																																						
Y4 =																																																																																																						
Y5 =																																																																																																						
Y6 =																																																																																																						
Y7 =																																																																																																						
X	Y1																																																																																																					
0	1																																																																																																					
1	0																																																																																																					
2	-1																																																																																																					
3	0																																																																																																					
4	1																																																																																																					
5	0																																																																																																					
6	-1																																																																																																					
X	Y1																																																																																																					
57	-7E-13																																																																																																					
58	-1																																																																																																					
59	9E-13																																																																																																					
60	1																																																																																																					
61	-1E-12																																																																																																					
62	-1																																																																																																					
63	1E-12																																																																																																					

Exercice 2 (Suites récurrentes)

1)
$$\begin{cases} u_0 = 0 \\ u_{n+1} = 4u_n - 2 \end{cases}$$

• $u_0 = \boxed{0}$;

• $u_1 = 4u_0 - 2 = 4 \times 0 - 2 = 0 - 2 = \boxed{-2}$;

• $u_2 = 4u_1 - 2 = 4 \times (-2) - 2 = -8 - 2 = \boxed{-10}$;

• $u_3 = 4u_2 - 2 = 4 \times (-10) - 2 = -40 - 2 = \boxed{-42}$.

Vérification à la calculatrice :

0	0
4*Ans-2	-2
4*Ans-2	-10
█	

4*Ans-2	-42
4*Ans-2	-170
4*Ans-2	-682
█	

2)
$$\begin{cases} u_0 = -1 \\ u_{n+1} = 2 + u_n^2 \end{cases}$$

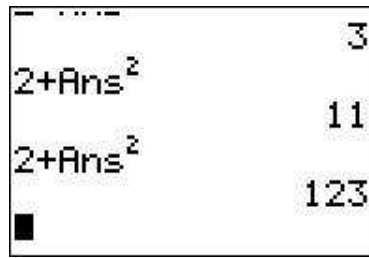
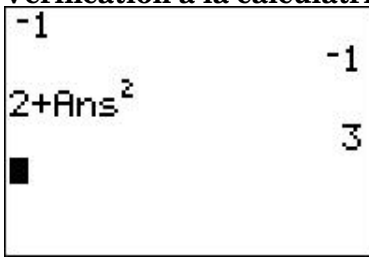
• $u_0 = \boxed{-1}$;

• $u_1 = 2 + u_0^2 = 2 + (-1)^2 = 2 + 1 = \boxed{3}$;

• $u_2 = 2 + u_1^2 = 2 + 3^2 = 2 + 9 = \boxed{11}$;

• $u_3 = 2 + u_2^2 = 2 + 11^2 = 2 + 121 = \boxed{123}$.

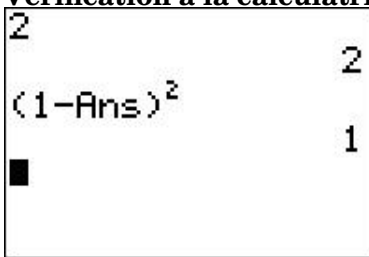
Vérification à la calculatrice :



3)
$$\begin{cases} u_0 = 2 \\ u_{n+1} = (1 - u_n)^2 \end{cases}$$

- $u_0 = \boxed{2}$;
- $u_1 = (1 - u_0)^2 = (1 - 2)^2 = (-1)^2 = \boxed{1}$;
- $u_2 = (1 - u_1)^2 = (1 - 1)^2 = 0^2 = \boxed{0}$;
- $u_3 = (1 - u_2)^2 = (1 - 0)^2 = 1^2 = \boxed{1}$.

Vérification à la calculatrice :



4)
$$\begin{cases} u_0 = 5 \\ u_{n+1} = \frac{2u_n}{3u_n + 1} \end{cases}$$

- $u_0 = \boxed{5}$;
- $u_1 = \frac{2u_0}{3u_0 + 1} = \frac{2 \times 5}{3 \times 5 + 1} = \frac{10}{15 + 1} = \frac{10}{16} = \frac{5}{8} = 0,625$;
- $u_2 = \frac{2u_1}{3u_1 + 1} = \frac{2 \times \frac{5}{8}}{3 \times \frac{5}{8} + 1} = \frac{\frac{10}{8}}{\frac{15}{8} + \frac{8}{8}} = \frac{10}{23} = \frac{10}{8} \times \frac{8}{23} = \frac{10}{23} \approx 0,435$;
- $u_3 = \frac{2u_2}{3u_2 + 1} = \frac{2 \times \frac{10}{23}}{3 \times \frac{10}{23} + 1} = \frac{\frac{20}{23}}{\frac{30}{23} + \frac{23}{23}} = \frac{20}{53} = \frac{20}{23} \times \frac{23}{53} = \frac{20}{53} \approx 0,377$.

Vérification à la calculatrice :

